Augmented Box Model for Colliding Turbidity Currents

Conor Olive*, Nathan Willis, François Blanchette

Dept. of Applied Mathematics, University of California Merced

*conorolive.com

Motivation

- **Deep-sea mining** is a burgeoning industry that could supply rare metals [3]
- The resulting environmental impacts remain poorly understood
- As vehicles scour the ocean floor, they generate **turbidity currents** that will inevitably collide
- Complex models like Navier-Stokes or Shallow-Water equations (SWE) accurately model the spread of surface plumes, but are slow
- We seek a simpler model that still captures collision dynamics between currents
- We create this model by width-averaging solutions of SWE and using this as training data for the system identification algorithm known as **SINDy** (Sparse Identification of Nonlinear Dynamics) [2]

Box Model

- The standard box model is [4]

$$\frac{dx_b}{dt} = \mathrm{Fr}\left[\frac{V\,c_b}{x_b(t)}\right]^{1/2} \quad (1)$$

$$\frac{dc_b}{dt} = -\frac{u_s c_b x_b(t)}{V} \quad (2)$$

where x_b represents the position of the current front and c_b represents its concentration

- The constants V, u_s, Fr are volume, settling speed, and Froude number, respectively
- Multiple currents may be simulated concurrently, but there is **no known way to have them interact**
- Progression of the system with two non-interacting currents is shown in Fig 1

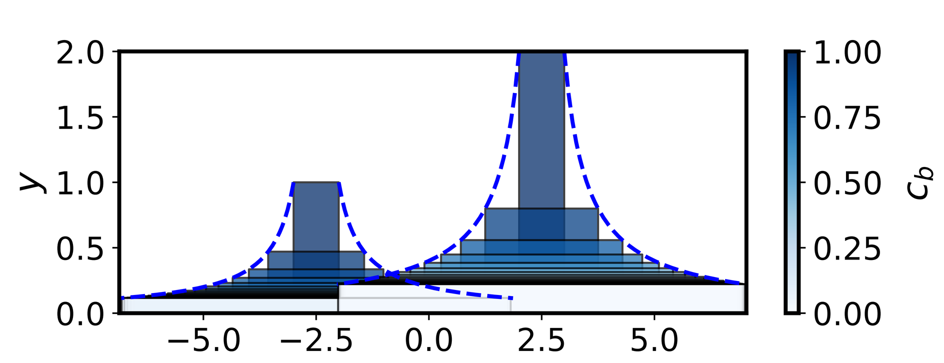

Fig. 1: Current progression of a box-model system shown at 0.5 time intervals. Color indicates particle concentration, and y indicates current height.

Shallow–Water Equations

- The Shallow–Water Equations (SWE) are derived from Navier–Stokes. Unlike the Box Model, are averaged over just height, and not width [4]
- The SWE system we use to generate our training data is [1]

$$\frac{\partial h}{\partial t} + \nabla\cdot q = \frac{1}{\mathrm{Re}}\nabla_h^2 h \quad (3)$$

$$\frac{\partial q}{\partial t} + \nabla\cdot\left(\frac{q^2}{h}\right) = \frac{1}{\mathrm{Re}}\nabla_h^2 q - \nabla\left[h\left(\phi_1+\phi_2\right)\right] \quad (4)$$

$$\frac{\partial \phi_i}{\partial t} + \nabla\cdot\left(\phi_i\frac{q}{h}\right) + u_s\frac{\phi_i}{h} = \frac{1}{\mathrm{Pe}}\nabla_h^2\phi_i, \qquad \text{for } i = 1,2 \quad (5)$$

where h is the height of the current, q is velocity times height, and ϕ is concentration times height

- The Reynolds number Re, Péclect number Pe, and settling speed u_s are defined as constants
- We numerically solve (3) - (5) using a finite–volume method
- Unlike the Box Model, Shallow–Water Equations allow for collision between currents, as shown in Fig. 2

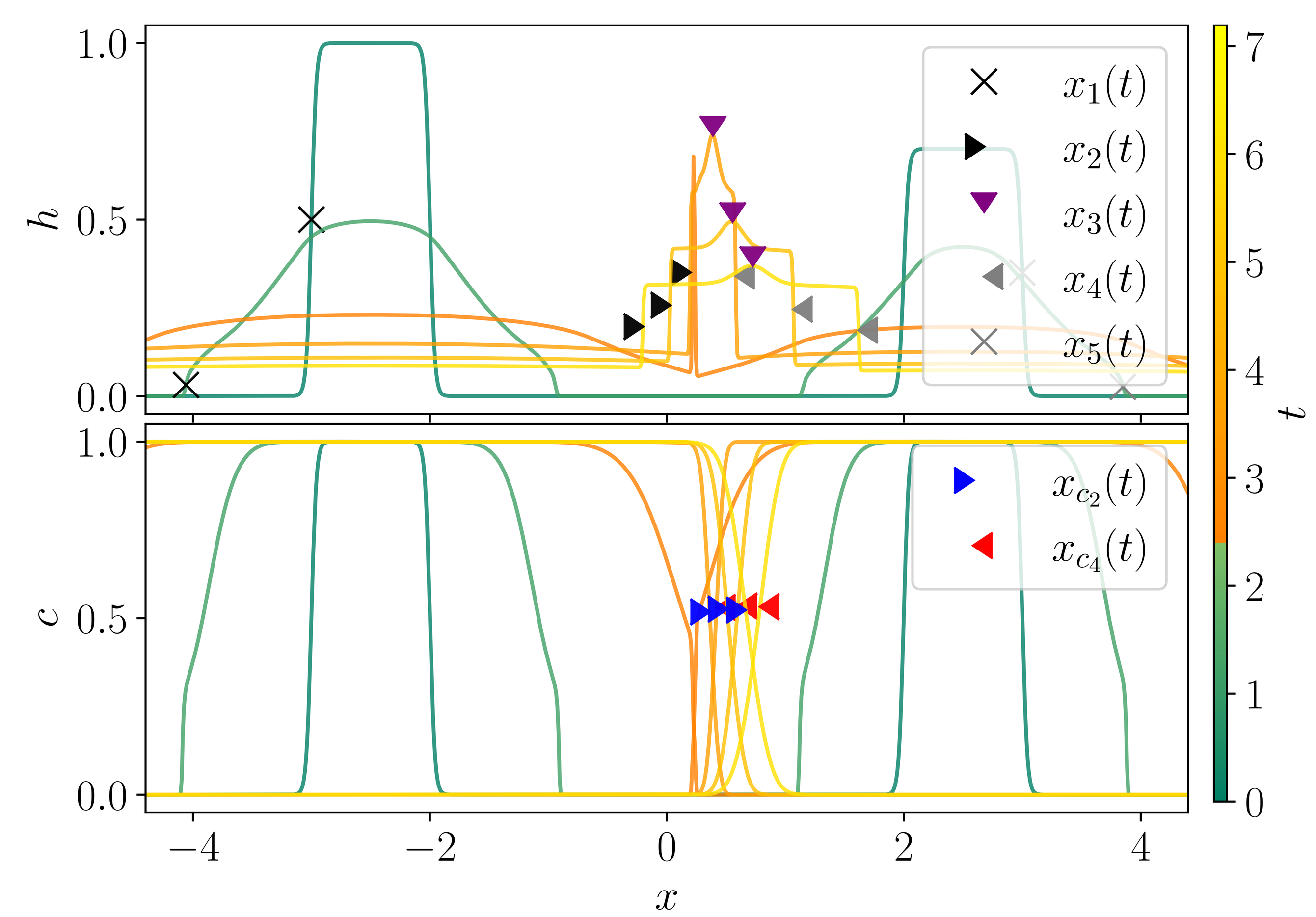

Fig. 2: Progression of a collision in a SWE system; green is before collision and orange is after collision.

"Boxifying" Method

- We width-average or "boxify" solutions to SWE for system identification
- The primary challenge is identifying the left and right fronts from the collision, which appear as shocks or reflected bores x_2 and x_4 in Fig. 2

Box-Generating Method

1. At each time t, locate inflection points in the concentration x_{c_2} and x_{c_4}, near the collision point (see Fig. 2) and defined the current center $x_3(t)$ as
$$x_3(t) = \frac{1}{2}\left(x_{c_2}(t) + x_{c_4}(t)\right).$$
2. At each time t, identify inflection points in the height x_1, x_2, x_3, x_4, x_5 such that $h''(x_n(t), t) = 0$ with $x_n < x_{n+1}$.

"Boxifying" Method (Cont.)

3. Define current width $w_n = x_{n+1} - x_n$ and averages of height, h_n, and concentration c_n, by integrating between adjacent fronts e.g.
$$h_n(t) = \frac{1}{w_n}\int_{x_n}^{x_{n+1}} h(x,t)\,dx.$$
Volumes are then defined as $V_n = h_n w_n$ for $n = 1, 2, 3, 4$.

Boxified Result

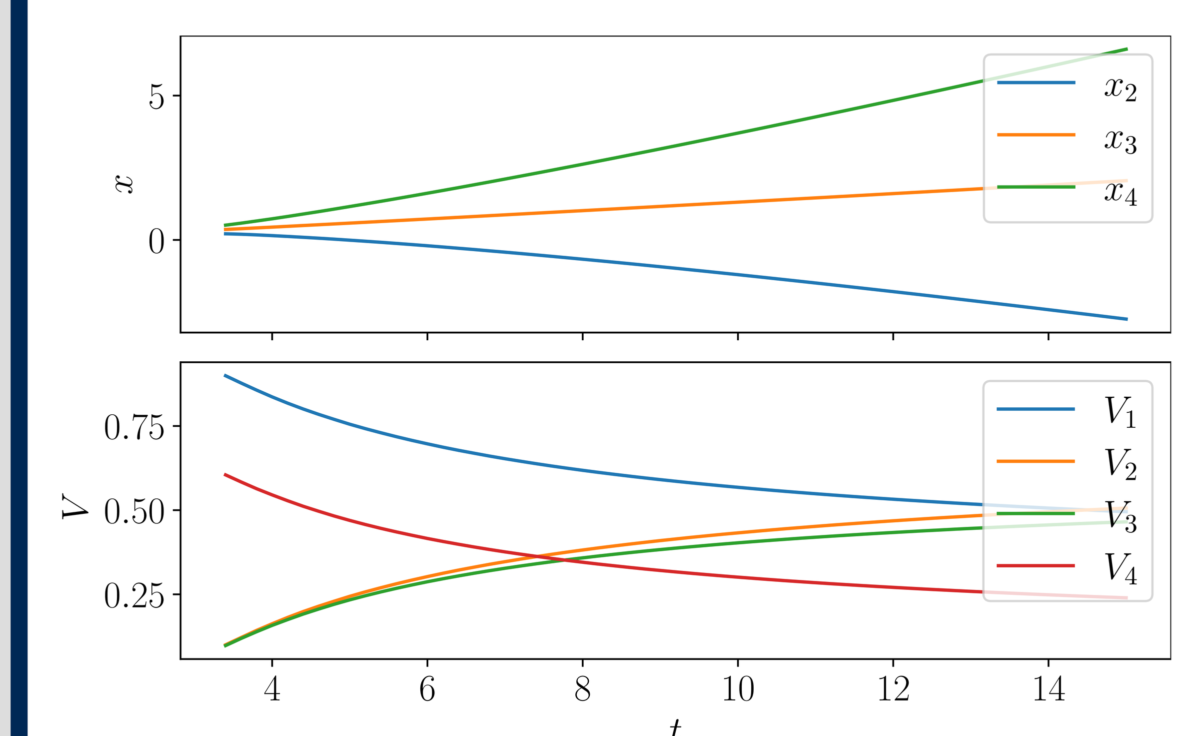

Fig. 3: Width-averaged quantities in a SWE collision between two currents shown in Fig 2.

System Identification with SINDy

- We attempt to identify three separate systems to avoid regressing on a highly underdetermined system
- Using SINDy, the following system describes the inner fronts, identified from a first degree polynomial library containing terms V_n and c_n

$$(x_2)' = 0.48V_1 - 0.14V_2 - 0.31V_3 - 0.49V_4 - 0.07c_2 - 0.10c_3 - 0.06c_4$$

$$(x_3)' = 0.51V_1 - 0.00V_3 - 0.54V_4 - 0.01c_1 + 0.07c_2 - 0.03c_4$$

$$(x_4)' = 0.49V_1 + 0.11V_2 + 0.36V_3 - 0.54V_4 - 0.19c_1 + 0.33c_2 - 0.08c_3 + 0.21c_4$$

Identified Systems

- We obtain good agreements over the range chosen

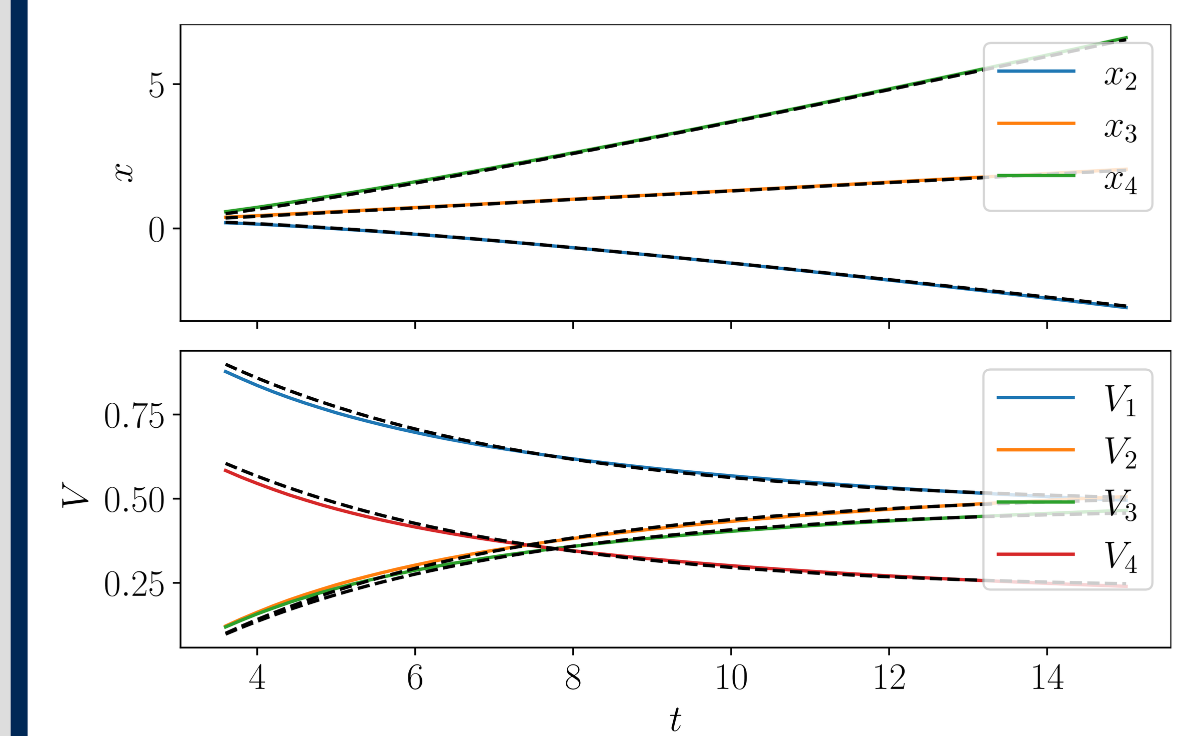

Fig. 4: Comparison of the identified system. Identified system shown as dashed lines.

Discussion

- Current results recover time evolution well but are not easy to interpret physically
- We aim to obtain physically interpretable coefficients through careful library selection
- A well chosen library should be able to recover and extend the usual box model

References

[1] F. Blanchette. "Shallow-water equations and box model simulations of turbidity currents from a moving source". In: *Phys. Rev. Fluids* 7 (8 Aug. 2022), p. 084301.

[2] S. Brunton, J. Proctor, and N. Kutz. "Discovering governing equations from data by sparse identification of nonlinear dynamical systems". eng. In: *Proceedings of the National Academy of Sciences - PNAS* 113.15 (2016), pp. 3932–3937. ISSN: 0027-8424.

[3] T. Peacock and R. Ouillon. "The Fluid Mechanics of Deep-Sea Mining". In: *Annual Review of Fluid Mechanics* 55.1 (2023), pp. 403–430.

[4] M. Ungarish. *An Introduction to Gravity Currents and Intrusions*. 2019.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. (DMS-1840265)