# Box Model Simulations of Turbidity Currents

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## Introduction

- Deep-sea mining is a burgeoning industry which could supply rare metals
- Environmental impact of deep-sea mining remains poorly understood
- Complex models like Navier-Stokes or Shallow-Water equations (SWE) accurately model the spread of surface plumes, but take many hours, days, or weeks to solve
- We seek a simpler model that may be solved in seconds such that the parameter space may be explored easily
- For this we use a custom version of the "box model" which averages quantities horizontally and vertically

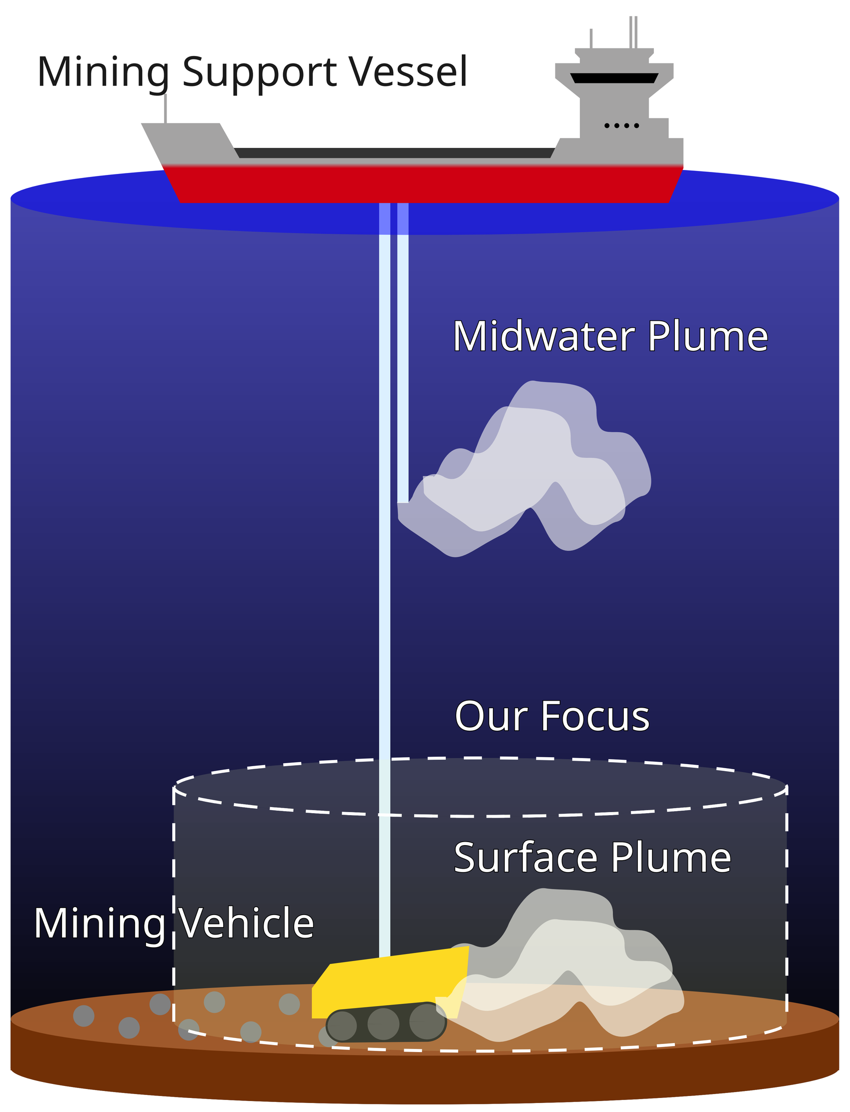


Fig. 1: Polymetallic nodule mining

## Methods

- We start with a box of volume $V$ (Figure 2)

$$x_N(t)h(t) = V \quad (1)$$

- From Ungarish 2019 the ordinary box model is

$$\frac{dx_N(t)}{dt} = \mathrm{Fr}\,[h(t)c_b]^{1/2} \quad (2)$$

- where the Froude number (Fr) is defined as

$$\mathrm{Fr} = \frac{u}{\sqrt{g' L c_b}} \quad (3)$$

- where $u$ is flow velocity, $g'$ is reduced gravity
- The concentration of sediment $c_b$ in the current drops out as particles deposit on the seafloor

## Methods (cont.)

$$\frac{dc_b}{dt} = -\frac{u_s c_b}{h(t)} \quad (4)$$

- where $u_s$ is the particle settling speed

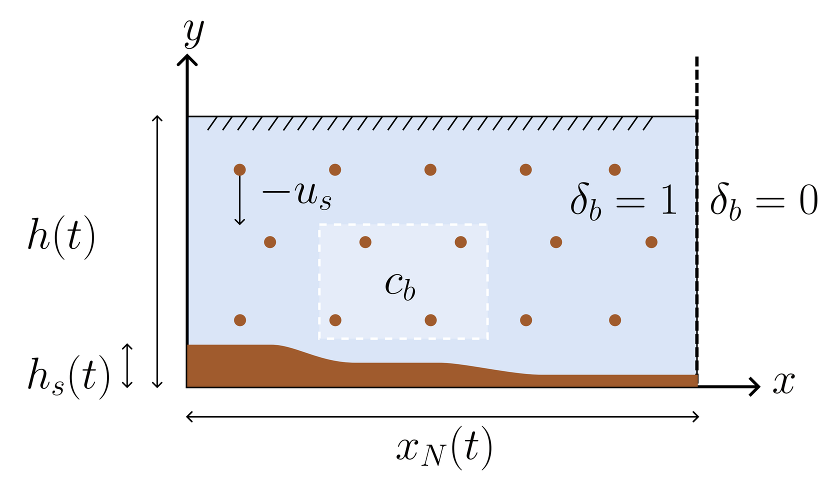


Fig. 2: The physical system described in (1)-(4)

### The Box Model

- From (1)-(4) we get a system of ODEs to describe our turbidity current

$$\frac{dx_N}{dt} = \mathrm{Fr}\left[\frac{Vc_b}{x_N(t)}\right]^{1/2} \quad (5)$$

$$\frac{dc_b}{dt} = -\frac{u_s\,c_b\,x_N(t)}{V} \quad (6)$$

- We also wish to recover from our numerical solution the deposited sediment height $h_s$

$$\frac{dh_s}{dt} = -\frac{c_b u_s \delta_b}{h(t)} \quad (7)$$

- where $\delta_b(x,t)$ is an indicator function which is defined as 0 at locations outside the box and 1 inside the box (Figure 2)
- We solve (5)-(6) using a fourth-order Runge-Kutta method with fifth-order error prediction (Dormand and Prince 1980)
- This solver is implemented in SciPy as `scipy.integrate.RK45()` (Virtanen et al. 2020)
- Validation of the order of the solver is shown in Figure 3
- We may be unsure of time scales, so we define function that automatically terminates the solver when all current concentrations drop below 0.005

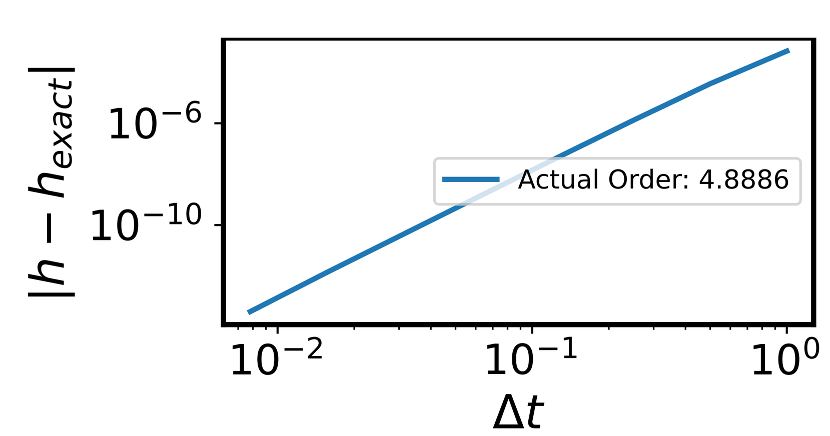


Fig. 3: Order of the solver used on (5)-(6)

## Results

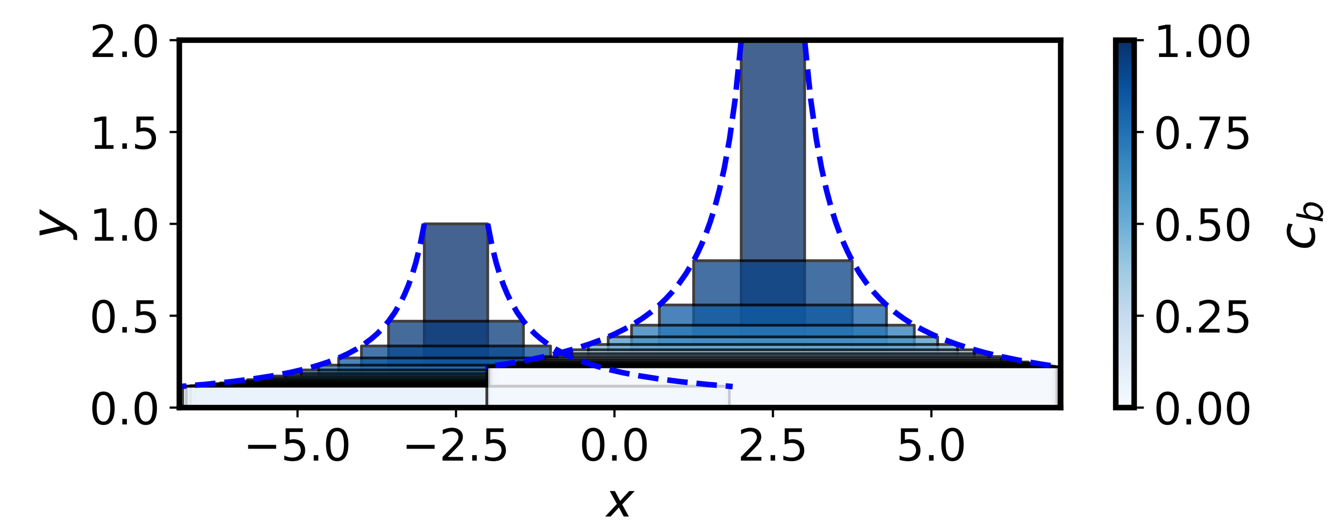


Fig. 4: Current progression shown at 0.5 time intervals, where color indicates remaining particle concentration

- Our resulting model accurately reflects behavior of real turbidity currents;
- The box model spreads out quickly at first, but eventually slows down as the concentration of sediment and pressure gradient of the current approach zero
- Figure 4 shows this behavior where each box represents 0.5 change in time

## Effect of Settling Speed

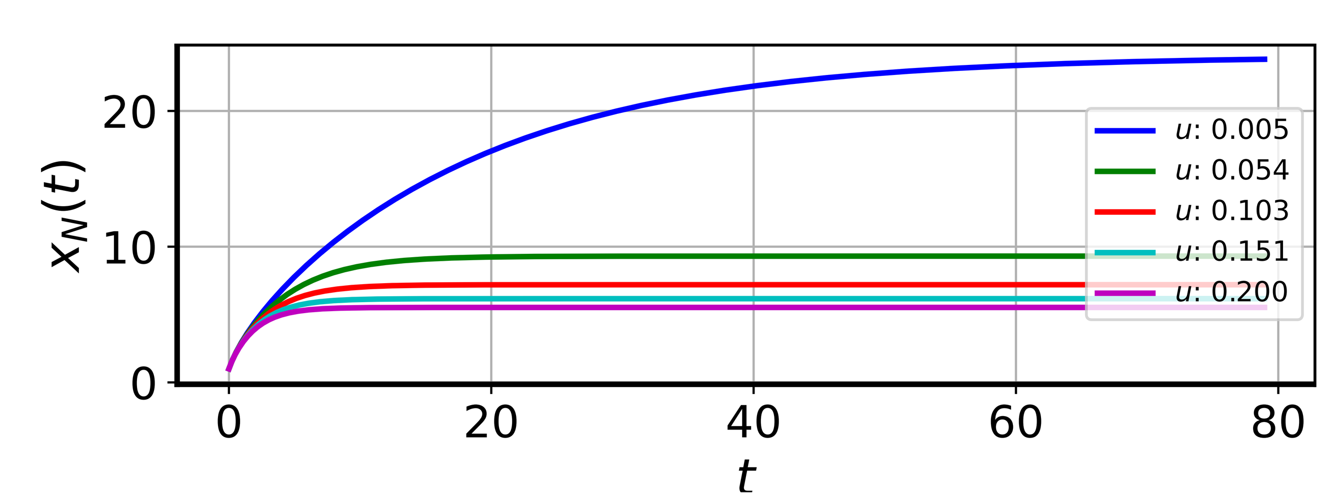


Fig. 5: Current width for various $u_s$

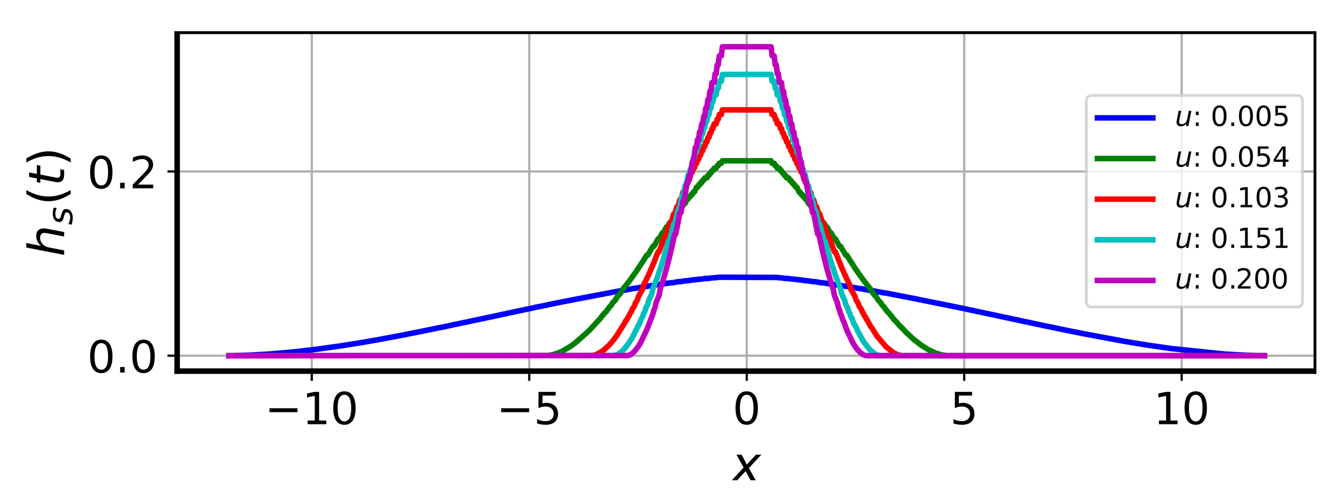


Fig. 6: Deposits at $t \approx 78.9$

- Qualities such as the settling speed of sediment in the deep sea are unknown, but we may use our model to explore this parameter space
- Our initial concentration is 1 and our initial volume is also 1
- We keep all quantities fixed besides $u_s$
- Our model shows that larger settling speeds will cause sediment to deposit (and concentration to decrease) more quickly
- Since concentration decreases faster with larger settling speeds, the final width of the current and total breadth of the deposit is smaller

## Effect of Froude Number

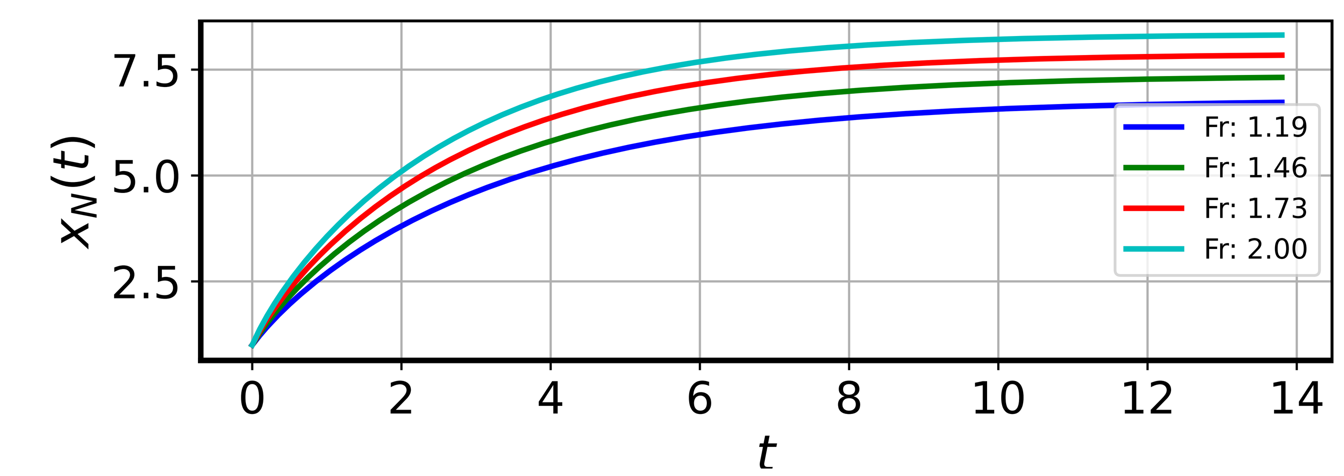


Fig. 7: Final width for various Froude numbers, where settling speed is $u_s = 0.1$

- We also investigate the expected range of values for the Froude numbers
- Expected real-world values of Fr vary only slightly (approximately 1.19 - 2)
- Larger Froude numbers increase the final width of the deposits, as expected from (3)

## Discussion

- It is known (Blanchette 2022) that the box model, while simpler, still closely approximates SWE enough to make meaningful predictions
- We have also shown that the box model retains important characteristics of higher-dimension models such as the relationship between current, deposit spreading, and parameters like settling speed $u_s$ and the Froude number
- The box model does not consider interactions of multiple turbidity currents, such as is possible with SWE
- In future research, we hope to accurately approximate these dynamics within the lower-dimension box model using SINDy or analogous methods

## References

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